Fluid Mechanics and Dynamics of Gases

Dr. Nancy Moore
We are grateful to NCEES for granting us permission to copy short sections from the FE Handbook to show students how to use Handbook information in solving problems. This information will normally appear in these videos as white boxes.
### Other Disciplines FE Specifications

<table>
<thead>
<tr>
<th>Topic: Fluid Mechanics and Dynamics of Gases 4-6 FE exam problems</th>
<th>Exam Problem Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Fluid properties (e.g., ideal and non-ideal gases)</td>
<td>21, 26, 81, 97</td>
</tr>
<tr>
<td>B. Dimensionless numbers (e.g., Reynolds number, Mach number)</td>
<td></td>
</tr>
<tr>
<td>C. Laminar and turbulent flow</td>
<td></td>
</tr>
<tr>
<td>D. Fluid statics</td>
<td></td>
</tr>
<tr>
<td>E. Energy, impulse, and momentum equations</td>
<td></td>
</tr>
<tr>
<td>F. Duct and pipe flow and friction losses</td>
<td></td>
</tr>
<tr>
<td>G. Fluid transport systems (e.g., series and parallel operations)</td>
<td></td>
</tr>
<tr>
<td>H. Flow measurement</td>
<td>28</td>
</tr>
<tr>
<td>I. Turbomachinery (e.g., fans, compressors, turbines)</td>
<td>84</td>
</tr>
</tbody>
</table>
1. Nitrogen gas has a temperature of 127°C when the pressure and volume are 100 kPa and 0.3 m³, respectively. The mass is most nearly, in kg,

(A) 0.25  (B) 0.8  (C) 1.25  (D) 4.0

\[ PV = mRT \]

\[ m = \frac{(100 \text{ kPa})(0.3 \text{ m}^3)}{\left(0.2968 \frac{kJ}{kgK}\right)(127 + 273)K} = 0.253 \text{ kg} \]
2. An ideal gas initially at 100°C, 1 atm undergoes two processes. First, the volume is held constant until the pressure doubles. Second, the pressure is held constant until the volume is reduced to one-third of the original volume. The final temperature of the gas is, in °C,

(A) -24  (B) 66  (C) 100  (D) 248

\[ P_1 V_1 = P_2 V_2 = P_3 V_3 \]

\[ \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \]

\[ T_3 = \frac{2}{3} T_1 = \frac{2}{3} (100 + 273) K = 248 K = -24 ^\circ C \]
3. The velocity of sound, $c$, in an ideal gas can be found from $c = \sqrt{kRT}$. Which of the following will be an alternate form of the equation?

(A) $c = \sqrt{\frac{kP}{v}}$  
(B) $c = \sqrt{\frac{k}{P}}$  
(C) $c = \sqrt{\frac{kP}{\rho}}$  
(D) $c = \sqrt{\frac{\rho}{P}}$

**COMPRESSIBLE FLOW**

Mach Number

The local *speed of sound* in an ideal gas is given by:

$c = \sqrt{kRT}$, where

$c =$ local speed of sound

$k =$ ratio of specific heats $= \frac{c_p}{c_v}$

$R =$ specific gas constant $= \bar{R}/$(molecular weight)

$T =$ absolute temperature

example: speed of sound in dry air at 1 atm 20°C is 343.2 m/s.
4. The air in an automobile tire has a gage pressure of 200 kPa when its temperature is 25°C. After being driven for 2 hours, the tire is found to have a temperature of 120°C. If the volume of the tire stays constant, what will be the new gage pressure of the air, in kPa? Assume atmospheric pressure is 100 kPa.

(A) 286     (B) 296     (C) 306     (D) 316

\[ P_{2,a} = P_{1,a} \frac{T_2}{T_1} = \frac{(200 + 100)kPa \cdot (120 + 273)K}{(25 + 273)K} = 395.6 \text{ kPa} \]

\[ P_{2,g} = P_{2,a} - P = 295.6 \text{ kPa} \]
5. Air is flowing through a duct at a velocity of 500 ft/s at 70°F. Use the following data:

Specific heat of air at constant pressure= 6,000 ft-lb/slug-°R
Specific heat of air at constant volume= 4,285 ft-lb/slug-°R
Gas Constant= 1,715 ft²/s²°R

Assuming air to be an ideal gas, the Mach number under the above conditions is most nearly

(A) 0.37 \hspace{1cm} (C) 0.44
(B) 1.32 \hspace{1cm} (D) 2.25

\[
c = \sqrt{kRT} = \sqrt{\frac{c_p}{c_v}}RT = \sqrt{\frac{(6000)}{(4285)}(1715)(70 + 460)°R} = 1128.2
\]

\[
Ma = \frac{V}{c} = 0.443
\]
6. Air is flowing through a 2 ft x 2 ft duct at a rate of 2,000 ft$^3$/s at 70ºF. The following data are available:

- Specific heat of air at constant pressure = 6,000 ft-lb/slug-ºR
- Specific heat of air at constant volume = 4,285 ft-lb/slug-ºR
- Gas Constant = 1,715 ft$^2$/s$^2$ºR

Assuming air to be an ideal gas, the Mach Number under the above conditions is most nearly

(A) 0.37  
(B) 1.32  
(C) 0.44  
(D) 2.25

\[ c = \sqrt{kRT} = \sqrt{\frac{c_p}{c_v}RT} = \sqrt{\frac{(6000)}{(4285)}(1715)(70 + 460)ºR} = 1128.2 \]

\[ V = \frac{Q}{A} = \frac{2000 \text{ ft}^3/\text{s}}{(2 \text{ ft})(2 \text{ ft})} = 500 \text{ ft/s} \]

\[ Ma = \frac{V}{c} = 0.443 \]
7. A vacuum gage connected to a chamber reads 35 kPa at a location where the atmospheric pressure is 92 kPa. Determine the absolute pressure in the chamber in kPa.

(A) 57  (B) 127  (C) 92  (D) 35

\[ P_{abs} = P_{atm} - P_{vac} = 57 \text{ kPa} \]
8. The atmospheric pressures at the top and the bottom of a building are read by a barometer to be 96.0 and 98.0 kPa. If the density of air is 1.0 kg/m³, the height of the building, in m, is

(A) 17    (B) 20    (C) 204    (D) 252

\[ P_2 - P_1 = \rho gh \]

\[ h = \frac{P_2 - P_1}{\rho g} = \frac{(98.0 - 96.0) \text{kPa}}{(1 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \frac{1000 \text{ Pa}}{1 \text{ kPa}} = 203.9 \text{ m} \]
9. Air with a density of 1.23 kg/m$^3$ and dynamic viscosity of 1.79e-5 Ns/m$^2$ flows through a 4.0 mm diameter drawn tubing with an average velocity of 5 m/s. Determine the head loss in a 0.1 m section of the tube in m.

(A) 1.48          (B) 2000          (C) 0.0015          (D) 0.30

**FLUID FLOW CHARACTERIZATION**

Reynolds Number
\[ Re = \frac{vDp}{\mu} = \frac{vD}{\nu} \]

\[ Re' = \frac{v^{(2-n)}D^n \rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{(n-1)}} \]

where
- \( \nu \) = fluid velocity
- \( \rho \) = the mass density
- \( D \) = the diameter of the pipe, dimension of the fluid streamline, or characteristic length
- \( \mu \) = the dynamic viscosity
- \( \nu \) = the kinematic viscosity
- \( Re \) = the Reynolds number (Newtonian fluid)
- \( Re' \) = the Reynolds number (Power law fluid)
- \( K \) and \( n \) are defined in the Stress, Pressure, and Viscosity section.

The critical Reynolds number \((Re)_c\) is defined to be the minimum Reynolds number at which a flow will turn turbulent.

Flow through a pipe is generally characterized as laminar for \( Re < 2,100 \) and fully turbulent for \( Re > 10,000 \), and transitional flow for \( 2,100 < Re < 10,000 \).
9. Air with a density of 1.23 kg/m$^3$ and dynamic viscosity of 1.79e-5 Ns/m$^2$ flows through a 4.0 mm diameter drawn tubing with an average velocity of 5 m/s. Determine the head loss in a 0.1 m section of the tube in m.

(A) 1.48 (B) 2000 (C) 0.0015 (D) 0.30

The Darcy-Weisbach equation is

$$h_f = f \frac{L}{D} \frac{V^2}{2g}, \text{ where}$$

- $f = f(Re, \varepsilon/D)$, the Moody, Darcy, or Stanton friction factor
- $D =$ diameter of the pipe
- $L =$ length over which the pressure drop occurs
- $\varepsilon =$ roughness factor for the pipe, and other symbols are defined as before

Reynolds number is

$$Re = \frac{\rho V D}{\mu} = \left(\frac{1.23 \text{ kg/m}^3}{5 \text{ m/s}}\right) \left(0.004 \text{ m}\right) = 1374$$

Therefore, the flow is laminar and $f = 64/Re$.

$$h_f = \frac{64 L V^2}{Re D 2g} = 1.48 \text{ m}$$
10. Air with a density of 1.23 kg/m$^3$ and dynamic viscosity of $1.79 \times 10^{-5}$ Ns/m$^2$ flows through a 4.0 mm diameter drawn tubing ($\varepsilon = .0015$ mm) with an average velocity of 50 m/s. Determine the head loss in a 0.1 m section of the tube in m.

(A) 1.7     (B) 890     (C) 178     (D) 89

$$Re = \frac{\rho V D}{\mu} = 13700$$

Therefore, the flow is turbulent and $f$ can be estimated with the Moody diagram.
10. Air with a density of 1.23 kg/m³ and dynamic viscosity of 1.79e-5 Ns/m² flows through a 4.0 mm diameter drawn tubing (ε = .0015 mm) with an average velocity of 50 m/s. Determine the head loss in a 0.1 m section of the tube in m.

(A) 1.7     (B) 890     (C) 178     (D) 89

\[ Re = \frac{\rho V D}{\mu} = 13700 \]

Therefore, the flow is turbulent and \( f \) can be estimated with the Moody diagram. \( \frac{\varepsilon}{D} = 0.000375 \) so \( f \approx 0.028. \)

\[ h_f = f \frac{L V^2}{D 2g} = 89.2 \text{ m} \]
11. Hot combustion gases (assumed to have the properties of air at room temperature) enter a gas turbine at 1 MPa and 1500 K at a rate of 0.1 kg/s, and exit at 0.2 MPa and 900 K. If heat is lost from the turbine to the surroundings at a rate of 15 kJ/s, the power output of the gas turbine, in kW, is

\( \dot{W} + \dot{Q} = \dot{m} c_p (T_1 - T_2) \)

\( \dot{W} = \left(0.1 \frac{kg}{s}\right) \left(1.0 \frac{kJ}{kgK}\right) (T_1 - T_2) - \dot{Q} = 45 \text{ kW} \)

(A) 15    (B) 30    (C) 45    (D) 60
12. In a heating system, cold outdoor air at 7ºC flowing at a rate of 4 kg/min is mixed adiabatically with heated air at 70ºC flowing at a rate of 3 kg/min. The exit temperature of the mixture, in ºC, is

(A) 34    (B) 39    (C) 63    (D) 77

\[ m_c p T_1 \text{ hot} + m_c p T_1 \text{ cold} = (m_\text{hot} + m_\text{cold}) c_p T_{mix} \]

where \( c_p = 1.00 \text{ kJ/kgK} \)

\[ T_{mix} = \frac{[mT_1]_{hot} + [mT_1]_{cold}}{(m_\text{hot} + m_\text{cold})} = 34 \]
13. Air enters a diffuser of a jet engine steadily. The air leaves with a velocity that is very small compared with the inlet velocity. Which of the following is true?

(A) The change in enthalpy is equal to the change in kinetic energy.
(B) The change in enthalpy is equal to the change in internal energy.
(C) The mass flow is not constant.
(D) The enthalpy is constant.

Assuming there is no heat or work,

\[ h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \]
14. The ventilating fan of the bathroom of a building has a volume flow rate of 30 L/s and runs continuously. If the density of air inside is 1.20 kg/m³, determine the mass of air vented out in one day, in kg.

\( \dot{m} = \rho Q = (1.20 \frac{kg}{m^3})(0.03 \frac{m^3}{s}) = 0.036 \frac{kg}{s} \)

\( m = \dot{m} \Delta t = (0.036 \frac{kg}{s})(24)(3600 \text{ s}) = 3110 \text{ kg} \)

(A) 864    (B) 0.036    (C) 311    (D) 3110
15. Consider two identical fans, one at sea level and the other on top of a high mountain, running at identical speeds. Which of the following is true?

(A) The mass flow rate of the two fans is the same.
(B) The volume flow rate of the two fans is the same.
(C) The volume flow rates are not constant.
(D) The density of air at each elevation is the same.

The density of air at sea level is higher than the density of air on top of a high mountain. Therefore, the volume flow rates of the two fans running at identical speeds will be the same, but the mass flow rate of the fan at sea level will be higher.
16. Carbon dioxide at a temperature of 0ºC and a pressure of 600 kPa flows through a horizontal 40 mm diameter pipe with an average velocity of 2 m/s. Determine the friction factor if the head loss is 30 m per 10 m length of pipe.

(A) 0.3   (B) 0.045   (C) 0.06   (D) 0.6

\[ h_f = f \frac{L v^2}{D 2g} \]

So the friction factor, \( f = 0.589 \)
17. Carbon dioxide at $20^\circ$C, a pressure of 550 kPa, and a viscosity of $1.4 \times 10^{-5}$ Ns/m$^2$ flows in a pipe at a rate of 0.04 N/s. Determine the maximum diameter in m allowed if the flow is to be turbulent.

(A) 0.18  (B) 1.7  (C) 0.04  (D) 0.55

The critical Reynolds number ($Re_c$), is defined to be the minimum Reynolds number at which a flow will turn turbulent.

Flow through a pipe is generally characterized as laminar for $Re < 2,100$ and fully turbulent for $Re > 10,000$, and transitional flow for $2,100 < Re < 10,000$.

\[
0.04 \frac{N}{s} = g \rho Q
\]

\[
Re = \frac{\rho V D}{\mu} > 2100
\]

\[
Re = \frac{4\rho Q}{\pi \mu D} = 2100
\]

\[
D = \frac{4\rho Q}{2100\pi \mu} = \frac{4(0.04 \frac{N}{s})}{2100\pi \mu} = 0.177 \text{ m}
\]
18. For air at a pressure of 200 kPa, temperature of 15ºC, and viscosity of 1.79 x 10^{-5} \text{Ns/m}^2, determine the maximum laminar volume flowrate for flow through a 2 cm diameter tube, in m$^3$/s.

(A) 0.0011  \hspace{1cm}  \text{(B) 0.00024} \hspace{1cm}  \text{(C) 0.024} \hspace{1cm}  \text{(D) 0.78}

\[
\rho = \frac{P}{RT} = \frac{200 \text{kPa}}{(0.287 \text{kJ/kgK})(15 + 273)K} = 2.42 \text{kg/m}^3
\]

\[
Re = \frac{\rho V D}{\mu} = 2100
\]

\[
V = \frac{2100 \left(1.79 \times 10^{-5} \text{Ns/m}^2\right)}{\left(2.42 \text{kg/m}^3\right)(0.02 \text{ m})} = 0.777 \text{m/s}
\]

\[
Q = AV = \frac{\pi}{4} D^2 V = 0.00024 \text{ m}^3/\text{s}
\]